ON SOME POSSIBILITIES OF STUDYING CHARACTERISTICS OF PERIODIC SIGNALS BASED ON CLUSTER ANALYSIS

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AIM • the estimation of the locus of periodic signals on the time axis, the degree of their presence in noise, etc.

THEORETICAL SUBSTANTIATION OF THE ESTIMATION METHOD AND PROBLEM STATEMENT

• Let a periodic signal be recorded at discrete moments of time \( t_1, ..., t_n \) \( (t_1=\Delta t = const, \ i=2, ..., N) \).

• Let us employ an estimation of a description of a signal on some running basis: \( \alpha_k = \sqrt{\frac{\sum_{j \in L} (x_j - M_k(L))^2}{L}} \); \( L \) is odd, \( x \) is the signal value, \( j=(L-1)/2, ..., N-(L-1)/2 \) and \( \alpha_k \) are integers. Thus, a grid \( Q=\Delta t h \) on a 2-D plane is juxtaposed with some signal: \( h=0, ..., \Delta t = \frac{L}{2}, ..., N-(L-1)/2 \). \( \sigma \) is the uppermost dispersion value and \( h \) is a threshold. Now, the problem can be reduced to investigating such grid for the estimation of signal presence on the time axis and the probability of its existence in noise.

• Let each point of the grid be represented by an event \( q_{ih} \): \( q_{ih}=1 \) if \( q_{ih} > \sigma \), else \( q_{ih}=0 \). Then a subset \( Q(h) \) of the adjacent instants will be called a cluster with the time interval \( \Delta t(h)=\Delta t q_{ih}+\Delta t q_{i+1,h} \) if \( q_{ih} q_{i+1,h} = 1 \). The cardinal number of such a cluster is \( \beta(h) = \sum_{i=0 \ (h_{ih} \in Q)} q_{ih} r = 1, ..., m(r,h) \). We can speak about two clusters of the two neighboring thresholds such that the cluster \( Q(h+1) \) is a child of \( Q(h) \) if they are intersecting in time: \( \Delta t(h) + \Delta t(h+1) \in (h) \). We will pool such clusters and call them a cluster family. The cardinal number of this cluster family is \( b(h) = \beta(h) + \sum_{i=(h+1)}^{(h)} \beta(h) (h+1) \), etc.

Let \( B_j = \sum_{h=0}^{h_j} b(h) \) be a common cardinal number of the \( j \)th cluster family on the grid \( Q \) ( \( h \) is the uppermost value of the threshold and \( h \) is the lowermost value of the threshold which limit the corresponding cluster family). Then the relation \( P_j = B_j / \sum_{j=1}^{34} B_j \) can be called a representative probability of the family \( Q \) (Znak 2009, Znak 2010).

TASK

The aim of the research affects the mode of analyzing signals.

DESCRIPTION OF THE ALGORITHM OF CLUSTER ANALYSIS

scanning the image of signal dispersion – picking out clusters sequences as functions of threshold values – construction of clusters families by using the fact of their intersections in time – estimation of probabilities of clusters families.

PROBABILITIES ESTIMATION

modes of estimation of representative probabilities will be presented using the notations “Weight” (W) and “Gradient” (G): \( W_i = \sum_{k=0}^{N_h} W_k (\Delta t - t_k^i + 1) / N_h \) \( W(h) = \sqrt{\sum_{k=0}^{N_h} W_k (\Delta t - t_k^i + 1)} \) \( W(h) = \sqrt{\sum_{k=0}^{N_h} W_k (\Delta t - t_k^i + 1)} \).

Now, estimations of the above probabilities of the family \( Q \) can be formulated in the following way (Znak 2015):

\[ P_j^1 = W_j / \sum_{j=1}^{34} W_j, \quad P_j^2 = W_j (h) / \sum_{j=1}^{34} W_j (h), \quad P_j^3 = G_j / \sum_{j=1}^{34} G_j. \]

EXPERIMENTS RESULTS

Introduction – explanation of results presentation: the use of a model of a noised periodic signal with the bandwidth of 7.2 + 8.2 Hz. – signal digitization frequency is \( \Delta t = 0.02 \) s. – white noise with the value of signal-to-noise ratio \( s/n=0.2 \) was used – the model length is 150 s. – the locus on a time axis of the source signal is 4 + 104 s.

OBTAINED DATA

<table>
<thead>
<tr>
<th>Case 1: ( L=13 ) s</th>
<th>Case 2: ( L=15 ) s</th>
<th>Case 3: ( L=17 ) s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{The quantity of depicted clusters families} )</td>
<td>34</td>
<td>37</td>
</tr>
<tr>
<td>( \text{The Estimated signal locus on a time axis} )</td>
<td>1.96 s + 104.2 s</td>
<td>1.98 s + 101.8 s</td>
</tr>
<tr>
<td>( \text{The representative probability} )</td>
<td>( P_{10} = 0.999774 )</td>
<td>( P_{10} = 0.997938 )</td>
</tr>
</tbody>
</table>

Discussion/conclusion – summary of the case, discussion and implication for further practice

an increase in accuracy of estimations of the signal locus on time axis can be achieved by calculating the detection rate of a signal (Znak 2009); the accuracy of estimating parameters and characteristics of periodic signals depends on the signal-to-noise ratio, therefore, for improving such a parameter with preservation of a signal wave form is proposed to attract the order filters for signal processing and statistical trials for selecting the qualitative filter project (Znak 2015, Znak 2010, Znak 2011); in the case of essentially small signal-to-noise ratio and in other complicated cases, the method of signal restoring can be used (Znak 2012); implications for further practice can be allocated in such fields of research as processing and analysis of geophysical data (Znak 2010), studying medical examinations data, etc.

Conclusion

• The aim of the research affects the mode of analyzing signals.
• The paper report starts with theoretical substantiation of the method of monitoring periodic signals and statement of the corresponding task.
• Description of the algorithm of monitoring periodic signals is presented.
• The characteristics of the model of a periodic signal used for demonstration of methodology are described.
• The results of the experiments carried out on demonstrated the dependence of the estimations quality on the value of running basis of dispersion.

REFERENCES


